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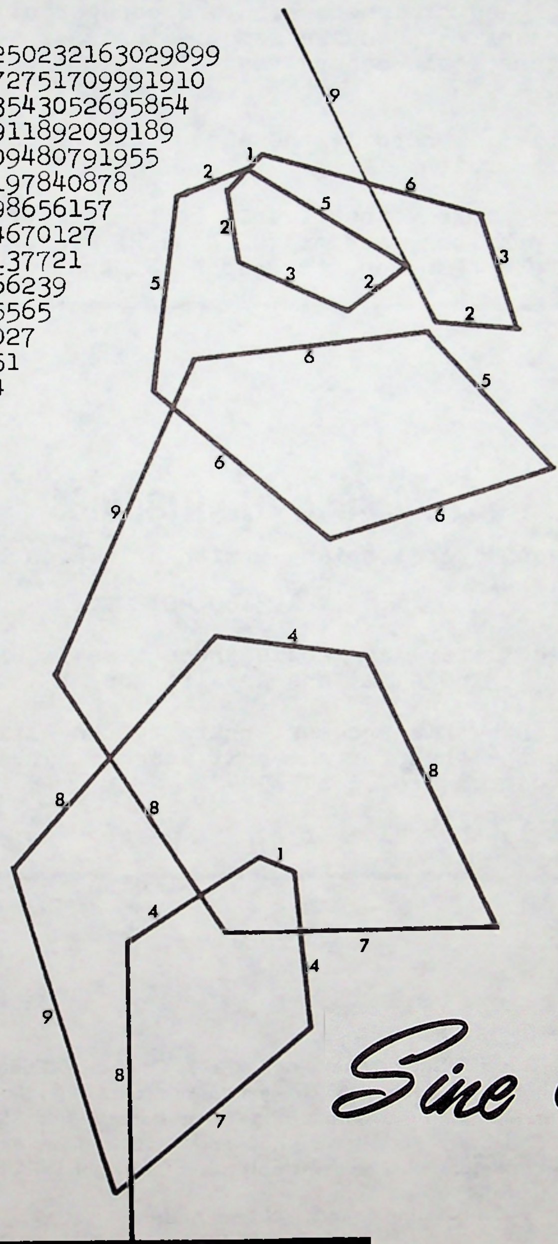

Sine Excursion

PROBLEM 37

SINE EXCURSION

A trip is begun at the origin, heading in the direction of the y-axis. Each leg of the trip has a length given by a digit of sine 1. 600 digits of this constant are shown on the cover. The angle between successive legs of the trip is one radian, taken clockwise. The digit zero in the decimal expansion of sine 1 indicates a leg of length zero and consequently a turn of two radians. A double zero indicates a turn of three radians; this occurs four times in the first 600 digits.

Problem: Where is the end of the 600th leg of the Sine Excursion?

Note: This is the fourth in the series of trips, for which previous entries were the Pi Dragon (PC6), the Road to e (PC8) and the Web of Fibonacci (PC10).

Sequence of Triangles

An equilateral triangle with unit area has an altitude, A_1, of

$$\sqrt[4]{3} = 1.3160740129\ldots$$

An equilateral triangle whose area is expressed by <u>that</u> number (1.316074...) has an altitude, A_2, 1.5098036...

Continue this process; that is, the altitude found at one stage is the area for the next stage. The next triangle has an altitude, A_3, of 1.6171144...

What is the value of A_{100}? PROBLEM 38

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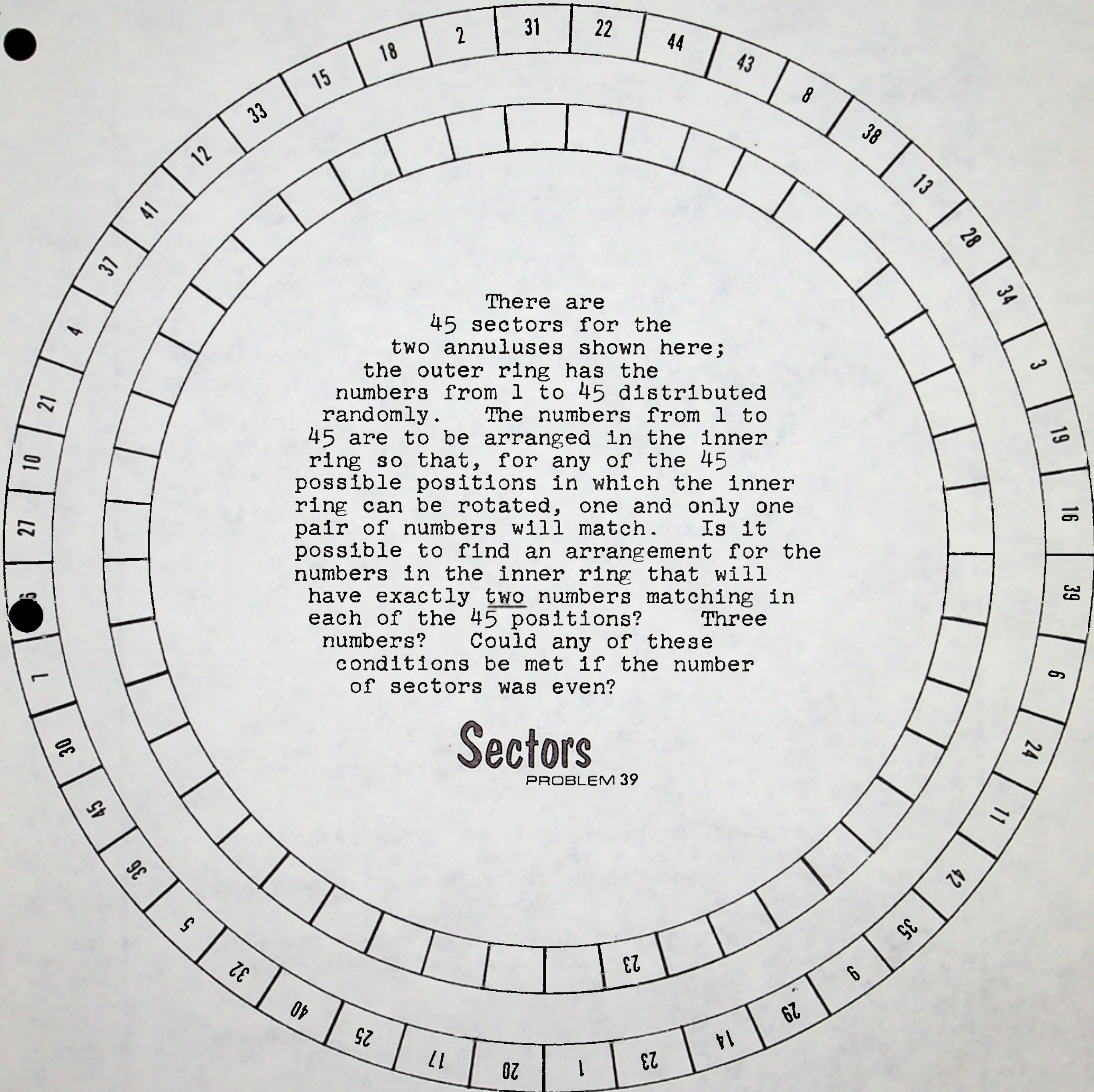


There are 45 sectors for the two annuluses shown here; the outer ring has the numbers from 1 to 45 distributed randomly. The numbers from 1 to 45 are to be arranged in the inner ring so that, for any of the 45 possible positions in which the inner ring can be rotated, one and only one pair of numbers will match. Is it possible to find an arrangement for the numbers in the inner ring that will have exactly <u>two</u> numbers matching in each of the 45 positions? Three numbers? Could any of these conditions be met if the number of sectors was even?

Sectors

PROBLEM 39

Pasta

The use of computers to play real games goes back now about 20 years. It is part of the Artificial Intelligence area, which is the attempt to simulate human decision-making (or creativity) by a computer program. Only in the areas of game playing and music composition has there been significant progress in AI.

The game of Kalah has been successfully programmed (that is, the computer program, playing against a human, usually wins). Fair success in checkers has been attained, and much attention has been paid to chess.

Zobrist, Albert, and Frederic Carlson, Jr., "An Advice-Taking Chess Computer," Scientific American, June, 1973, pp 93-105.

Mittman, Benjamin, "Can a Computer Beat Bobby Fischer?," Datamation, June, 1973, pp 84-87.

What does the term "real game" mean? It refers, first, to open-board games that have no random element; that is, all information is available to all players, and no luck is involved. The game must also be of sufficient complexity that it cannot be analyzed simply by exhaustion. Thus, Tic-Tac-Toe is not a real game in this sense, since every possible move can be readily enumerated (and stored in a computer). Chess is a real game; the standard estimate of the number of possible distinct chess games is 10^{120}. Games like Oware, Go, Fives (Go-Moku), and Pasta are real games.

The game of Pasta was first reported in the March 15, 1956 issue of Computing News. Attention is being called to it again on the grounds that it may constitute a supreme challenge for computer programmers. While the game itself is not as complex as chess, a program to implement a winning strategy may be much more complex. In fact, although Pasta has been around for 17 years, the only evidence that there is a strategy in playing it is that some people win consistently more than others.

Pasta is the invention of Alvin Paster. The name of the game comes by analogy to Laska, an invention of the chess master, Lasker.

The initial setup is that of checkers. The big difference in play is in jumping: the jumper adds the top checker of the jumpee to the bottom of his piece. A red piece, then, consists of a stack of checkers with any number of red checkers atop any number of (or no) black checkers. It is a red pawn, in the sense of checkers, if it consists of only one red checker atop any number of black checkers (which number includes zero). It is a red king, in the sense of checkers, if it consists of two or more red checkers atop any number of black

checkers (again, including zero black checkers). The object of the game is to place one of your pieces in the opponent's back row.

The moves are as in checkers: pawns can move only forward, kings can move in both directions, and jumps are compulsory. The board is oriented as in checkers (double corner on the right) and the usual rules of courtesy apply (e.g., when you move a piece, it stays moved). Black and red alternate in making the opening move; it is expedient in Pasta for a player to retain the same color through all games.

No checkers leave the board. The devastating feature of the game is that the pieces change both rank and color during play. There is one more rather important rule: you may not move into the opponent's back row (thus ending the game) if there is any jump available to you. Further, if a king jumps into the last row and it is possible for him to continue the jump out of the last row, he must do so, and the move does not constitute a win. This gives a useful last-ditch strategy.

Suppose your king jumps your opponent's king. You may not, as part of the same move, jump right back over his king and whittle it down to your own color (or to nothing at all). You may, however, as a result of a multiple jump, jump the same piece more than once. A trivial computation from the rules indicates that a stack can have only one red-black interface.

A king with three or more of its own color (on top, of course) is called a chiang. Such a piece is very powerful. It can move forward, displacing the opponent's men by forcing them to jump him at small expense to his own power. Something about the method of formation of chiangs seems to "bottle them up" in one's own back ranks; therefore, one may speak of unleashing the chiang.

A powerful set of last-ditch defensive plays revolve about forcing jumps or multiple jumps on the opponent, which effectively eliminate his threat by transporting the threatening piece away from one's own back rank.

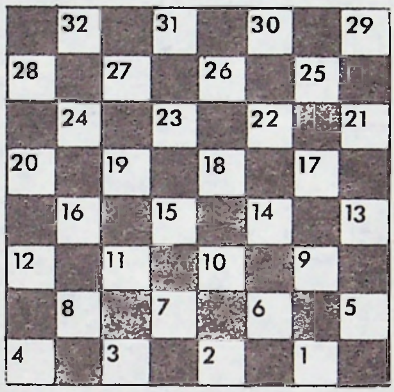
The game commonly opens with an exchange of jumps designed to produce kings. This has been variously characterized as "the dance of the wild loons," "the gooney-bird dance," and "yo-yo-ing." These jumps may be delayed for some time, but it seems inevitable that when one player starts it, the other must play along.

Games can be terrifically fast at times, taking as little as one minute, and as few as seven full moves. Innocuous looking jumps can create more jumps to form a totally unexpected chain reaction, running in some cases to as many as twenty forced moves. It is thus extremely difficult to see more than a few moves ahead.

The double corner seems to be the soft underbelly, and should be guarded carefully. Ability at checkers may be a drawback. In particular, many apparent exchanges don't "bounce" as in checkers, and long experience with checkers may be a boobytrap. Incidentally, it does not seem possible to have a draw in Pasta.

It is highly advisable to use a board with squares which are not the same color as the checkers. The Woolworth standard (red and black checkers; red and black board) creates confusion.

Following is a game which is quite common and which is a sort of "fool's mate." The board is numbered as in checkers, with red on squares 1-12:



Red opens 11-16. Then black 24-20; red 10-15; black 20-11; red 7-16; black 23-19. It is now safe for red to move 16-23, giving black a king. Since it seems foolhardy to give black a king, red tries 15-24 instead. This seems innocuous, but the following sequence yields a sudden loss for red: B 28-19; R 16-23; B 24-15; R 11-18; B 26-10; R 6-15; B 23-7; R 2-11; B 7-2. Most of these moves are forced.

An even shorter game is the following: B 24-19; R 11-16; B 27-24; R 16-20; B 22-18; R 20-17; B 31-24; R 10-15; B 18-11; R 8-15; B 19-10; R 11-18; B 23-14; R 6-31.

Both of these common games illustrate another unique feature of Pasta; namely, that it is frequently more proper to speak of one player losing than of the other player winning. The game offers one the opportunity, as in no other game, of carefully engineering your own defeat.

Speaking of Languages

ROBERT TEAGUE

This month's problem set is not as much a test of the programmer as it is of the Fortran compiler. In each of the three programs shown here, some unusual operations will take place that perhaps violate the spirit of the ANSI standard, but not the letter. The programs each present three problems for the programmer to answer for himself and then check his answers on the Fortran compiler. I would appreciate as many outputs as possible from different compilers as a test of how they individually interpret the standard; I will summarize the results in a future issue. Send the outputs to Speaking of Languages... c/o POPULAR COMPUTING.

The three questions concerning the programs are:

(1) Will it compile?

(2) Will it execute?

(3) What output will be produced?

I.

```
      DIMENSIØN L(3)
      EQUIVALENCE (J,L(2))
      L(2) = 10
      DØ 10 J = 1,3
      K = J
   10 CØNTINUE
      WRITE (6,20) L(2)
   20 FØRMAT (I7)
      STØP
      END
```

II.

```
      DIMENSIØN L(3)
      EQUIVALENCE (J,L(2))
      L(1) = 46
      L(2) = 47
      L(3) = 48
      DØ 10 J = 1,3
      I = J
      L(I) = J/2 + 4
   10 CØNTINUE
      WRITE (6,20) L(2)
   20 FØRMAT (I7)
      STØP
      END
```

III.

```
      DIMENSIØN L(3)
      EQUIVALENCE (J,L(2))
      L(1) = 5
      L(2) = 9
      L(3) = 3
      DØ 10 J = 1,3
      L(J) = J*4 + J/2
   10 CØNTINUE
      WRITE (6,20) L
   20 FØRMAT (3(2X,I7))
      STØP
      END
```

Note: Make appropriate changes to the WRITE statements in the programs if the printer is not assigned to unit 6 on your system.

Archimedes and the Value of Pi

by R. W. Hamming
Bell Laboratories, Murray Hill, New Jersey

In order to calculate the value of pi, Archimedes (287-212 BC) hit upon inscribing and circumscribing regular polygons in and around a circle. He argued that as the number of sides of the regular polygons was increased, the corresponding perimeters would approach each other and straddle the length of the circle. It would, perhaps, have been better to have used the areas rather than the lengths, but we will follow the general plan of Archimedes.

We propose, at each stage, to double the number of sides of the polygons. For this we need the corresponding formulas, using a circle of unit radius. From the Pythagorean Theorem we have, using S_n as the length of a side of an n-sided inscribed polygon,

$$\begin{aligned} \overline{OA}^2 + (S_n/2)^2 &= 1 \\ \overline{AB}^2 + (S_n/2)^2 &= S_{2n}^2 \\ \overline{OA} + \overline{AB} &= 1 \end{aligned}$$

Eliminate $\overline{AB}$:

$$\begin{aligned} \overline{OA}^2 &= 1 - (S_n/2)^2 \\ (1 - \overline{OA})^2 &= S_{2n}^2 - (S_n/2)^2 \end{aligned}$$

From the first of this pair:

$$\overline{OA} = (1/2)\sqrt{4 - S_n^2}$$

and, putting this in the second equation:

$$1 - \sqrt{4 - S_n^2} + 1 - (S_n/2)^2 = S_{2n}^2 - (S_n/2)^2$$

or:

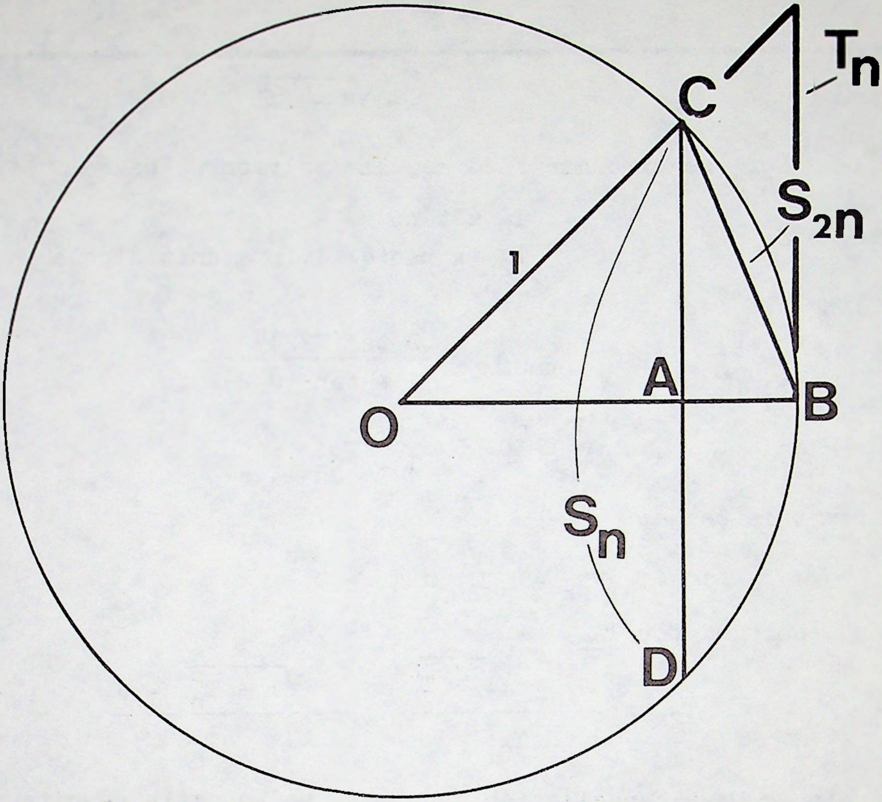
$$S_{2n} = \sqrt{2 - \sqrt{4 - S_n^2}}$$

But in this form as S_n approaches zero there is heavy cancellation, and hence roundoff errors, so we rewrite it:

$$S_{2n} = \sqrt{2 - \sqrt{4 - S_n^2}} \left[\frac{\sqrt{2 + \sqrt{4 - S_n^2}}}{\sqrt{2 + \sqrt{4 - S_n^2}}} \right]$$

Finally:

$$S_{2n} = \frac{S_n}{2 + \sqrt{4 - S_n^2}}$$



The total perimeter is $2nS_{2n}$ and this approaches 2π, as is evident in the top half of the accompanying table. The calculations are most convenient when we begin with an inscribed hexagon.

The formula

$$S_{2n}^2 = 2 - \sqrt{4 - S_n^2}$$

is a simple trigonometric identity in disguise.

$$\cos^2 \frac{\theta}{2} = \frac{1 - \cos\theta}{2}$$

$$1 - \sin^2 \frac{\theta}{2} = \frac{1 - \sqrt{1 - \sin^2\theta}}{2}$$

$$\sin^2 \frac{\theta}{2} = \frac{1 + \sqrt{1 - \sin^2\theta}}{2}$$

But $S_n = 2 \sin\theta$

$S_{2n} = 2 \sin(\theta/2)$

$$\frac{S_{2n}^2}{4} = \frac{1 - \sqrt{1 - (S_n/2)^2}}{2}$$

and hence $$S_{2n}^2 = 2 - \sqrt{4 - S_n^2}$$

For the circumscribed regular polygon we have

$$T_n = \tan\theta$$
$$T_{2n} = \tan(\theta/2) \text{ in a unit circle.}$$

Using the fact that

$$\tan\theta = \frac{2\tan(\theta/2)}{1 - \tan^2(\theta/2)}$$

$$T_n = \frac{2T_{2n}}{1 - T_{2n}^2}$$

from this we get

$$(T_n)T_{2n}^2 + 2T_{2n} - T_n = 0$$

and, solving for T_{2n},

$$T_{2n} = \frac{-1 \pm \sqrt{1 + T_n^2}}{T_n} = \frac{\sqrt{1 + T_n^2} - 1}{T_n}$$

Again we have cancellation problems, so we again rewrite it in the form

$$T_{2n} = \frac{\sqrt{1 + T_n^2} - 1}{T_n}\left[\frac{\sqrt{1 + T_n^2} + 1}{\sqrt{1 + T_n^2} + 1}\right] = \frac{T_n}{1 + \sqrt{1 + T_n^2}}$$

and again the total perimeter is $2nT_{2n}$ which approaches 2π as the number of sides is increased. For the case of circumscribed polygons, it is convenient to begin with a circumscribed square.

The accompanying table shows the calculations (carried out in double precision floating arithmetic in Fortran) extended through 19 doublings. Archimedes was able to show (doubling four times) that

$$3\ 1/7 > \pi > 3\ 10/71$$

that is, that pi lies between 3.1429 and 3.1408.

Exercise: Develop corresponding formulas for bracketing pi in terms of the <u>areas</u> of inscribed and circumscribed polygons.

N	S	P	
6	1.000000000000000000	6.0000000000000	Inscribed
12	.517638090197579003	6.2116570824292	
24	.261052384434151463	6.2652572265361	
48	.130806258457596413	6.2787004060810	
96	.065438165642262902	6.2820639017736	
192	.032723463252295914	6.2829049444990	
384	.016362279207442043	6.2831152157159	
768	.008181208052337751	6.2831677842187	
1536	.004090612582149333	6.2831809263444	
3072	.002045307360640435	6.2831842119340	
6144	.001022653814034129	6.2831850333605	
12288	.000511326923715387	6.2831852387171	
24576	.000255663463949531	6.2831852900563	
49152	.000127831732239428	6.2831853028620	
98304	.000063915866150637	6.2831853060052	
196608	.000031957933078956	6.2831853068201	
393216	.000015978966540842	6.2831853070529	
786432	.000007989483270284	6.2831853070529	
1572864	.000003994741635142	6.2831853071693	
3145728	.000001997370817571	6.2831853071693	
4	2.000000000000000000	8.0000000000000	Circumscribed
8	.828427124739391728	6.6274169979151	
16	.397824734754976816	6.3651957560796	
32	.196982806712185264	6.3034498147899	
64	.098253699537963256	6.2882367704296	
128	.049097244217591650	6.2844472598517	
256	.024544924758811248	6.2835007383255	
512	.012272000314987962	6.2832641613204	
1024	.006135942402761428	6.2832050204742	
2048	.003067963982175568	6.2831902354955	
4096	.001533981088687140	6.2831865391926	
8192	.000766990431529848	6.2831856150878	
16384	.000383495201667756	6.2831853841198	
32768	.000191747599070366	6.2831853263778	
65536	.000095873799316904	6.2831853119423	
131072	.000047936899631166	6.2831853083334	
262144	.000023968449811944	6.2831853074021	
524288	.000011984224905244	6.2831853071693	
1048576	.000005992112452622	6.2831853071693	
2097152	.000002996056226328	6.2831853071693	

(Two pi = 6.283185307179586476)

Tape Player Counters

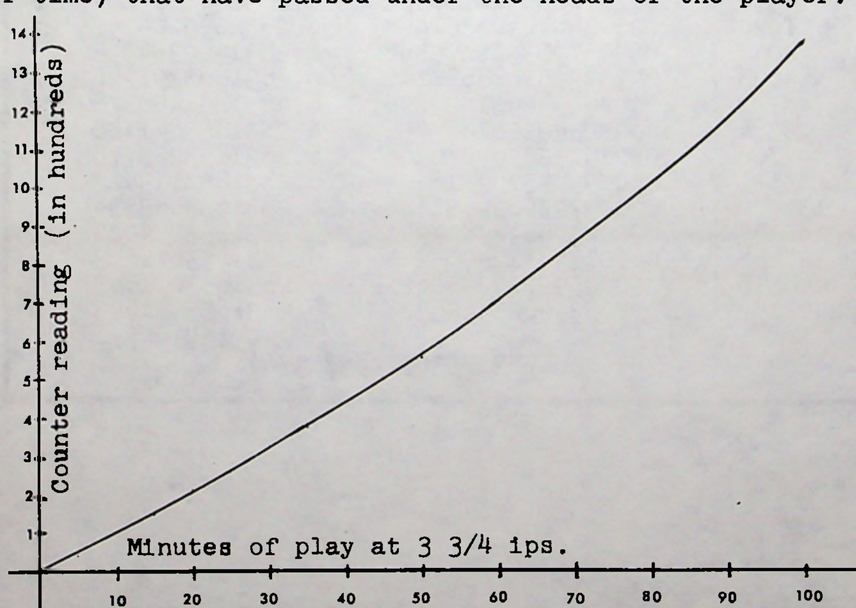
PROBLEM 40

Audio tape recorders usually have a counter (analog in its units position and digital in the higher order positions). It is difficult to say what these counters count. They are usually driven from the shaft of the supply reel, and they thus register some function of the number of turns of the supply reel. Their reading has no direct bearing on the amount of tape that passes the heads of the machine. When a tape is cued on one machine, the cue sheet is of little use on another machine--even one of the same brand. The graph shows the relation between a counter reading and the amount of tape passed under the head on a Sony 540.

The data for the curve shown is as follows:

Time	Counter	Time	Counter
5	51	55	635
10	103	60	704
15	157	65	775
20	211	70	849
25	267	75	926
30	324	80	1007
35	383	85	1092
40	443	90	1182
45	505	95	1278
50	569	100	1382

Similar data can be readily obtained for other machines, and other conditions of play. The problem that is involved is not strictly a computer problem, but since it may require curve fitting, it is a suitable problem for computists. So the Problem is: devise a formula by which tape player counter readings can be transformed into useful information; namely, the number of feet (or seconds of time) that have passed under the heads of the player.



Cubical Array

PROBLEM 41

Given a three-dimensional array, 10 x 10 x 10. The numbers from 1 to 1000 are to be put in this array in ascending order according to the distance of a cell from the origin. The shortest such distance is for the cell I = 1, J = 1, K = 1, for which the squared distance from the origin is given by

$$D^2 = I^2 + J^2 + K^2 = 3$$

and the greatest squared distance is for I = 10, J = 10, K = 10, which gives 300.

The six cells:

I	J	K
3	4	5
3	5	4
4	3	5
4	5	3
5	3	4
5	4	3

all have the same distance from the origin (taken as 0, 0, 0) and for such cases the ordering is to be taken as in the above list, with the values of I, J, and K taken as 3-digit numbers in ascending order.

Flowchart the logic for assigning the numbers from 1 to 1000 to the 1000 cells of the array. Write a Fortran program for the task.

Millions and Billions

Since computer people toss around the terms "million" and "billion" quite casually, it would help to be able to make these terms vivid.

Million is fairly easy. A million card chips occupy almost exactly three quarts. A million standard U.S. postage stamps would fill slightly over two cubic feet. A million seconds is 277.78 hours--about twice the Biblical time for the creation. A million IBM cards form a stack 556 feet high. A million days ago was 765 B.C. Your heart beats a million times every ten days, and a million breaths carries you through two months.

The concept of a billion is harder to visualize. In a billionth of a second, light travels 11.8 inches. A billion grains of common table salt weigh 300 pounds--about seven cubic feet. A backyard swimming pool contains nearly a billion drops. A billion seconds is 31.6888 years. Forty one average homes contain a billion cubic inches. If you watched "Sound of Music" 92,500 times, a billion feet of film would have gone through the projectors, and you would have spent 21 years of continuous viewing. However, a billion <u>frames</u> of film would have gone by in the first 482 days of viewing.

A round trip from Los Angeles to New York is a billion centimeters long. A quarter of a million copies of this issue of POPULAR COMPUTING would fill a billion cubic inches. A billion No. 18 paper clips weigh 911 tons. A billion of the staples that hold each copy of POPULAR COMPUTING together would weigh 39 tons, or as much weight as 42 Volkswagens. On the other hand, a billion Volkswagens would make (uncrushed) two million monuments of the volume of the pyramid of Khufu at Gizeh. And a billion copper pennies would weigh as much as 1100 Cadillacs.

A billion is close to the 30th power of 2, or the 19th power of 3. A billion standard pencils laid end to end would go around the earth nearly 5 times, and weigh 6168 tons. A billion 5-grain aspirin tablets weigh 417 tons and would fill your swimming pool eleven times. Those 13 billion hamburgers that McDonald's has sold would cover over 65 square miles. 5 1/2 sets of the <u>Encyclopedia Brittanica</u> contain a billion characters of printing, including commas. A billion grains of rice weigh over 50,000 pounds and fill 500 cubic feet--that is, a cube 7.96 feet on a side.

The majority of the computers in this country are <u>each</u> executing over a billion instructions every hour.

The Attached Tape

Attached to each copy of the initial press run of this issue of POPULAR COMPUTING is a tape recording containing a message of special interest to students.

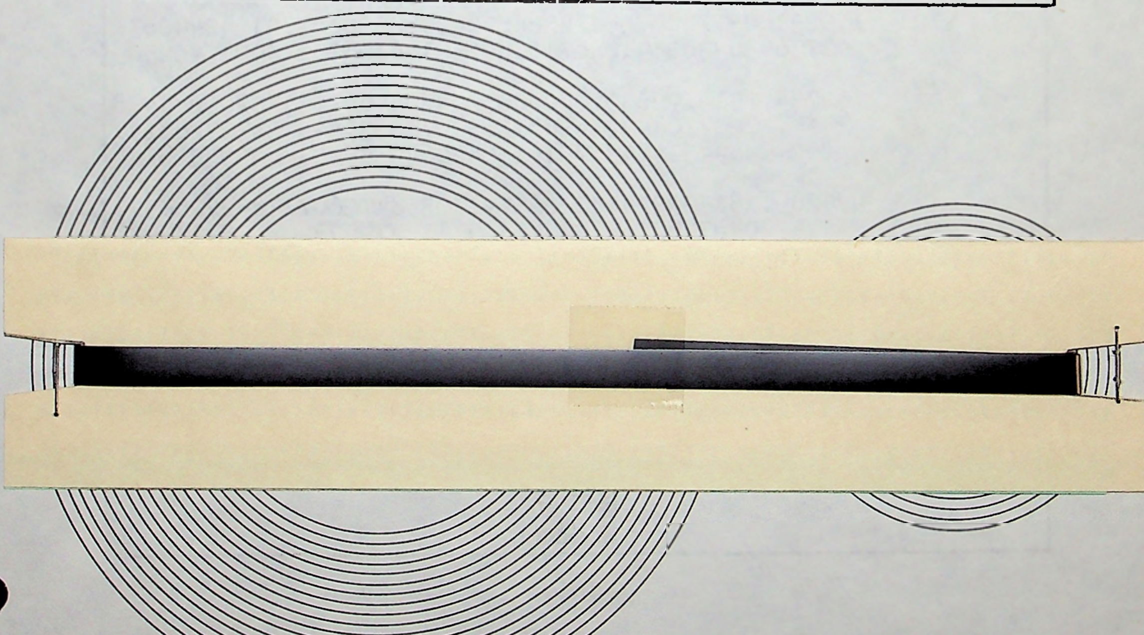
The recording is on 4-track tape, recorded at 3 3/4 inches per second in stereo.

The tape is wound on its bobbin with the head end out and with the recording surface in. The Scotch tape used to secure it to the cardboard should be cut off. There is a foot or so of blank tape at each end of the recording, but it will probably be necessary to attach some leader to both ends of the tape.

This is probably the first time that an audio message has been bound into a magazine.

The problems referred to on the tape are these:

8 (PC4-13) for the blanks from 38 up.
22 (PC8-14) for the DIS number PR6.
29 (PC9-16) for any three additional entries to the tables.
31 (PC10-9) for No. 4 of the four problems.
14 (PC6-10)
15 (PC7-1)
16 (PC7-10
19 (PC8-1)
21 (PC8-13)
24 (PC9-1)
25 (PC9-5)
26 (PC9-10)
27 (PC9-14)
30 (PC10-1)
32 (PC10-11)
33 (PC10-12)
34 (PC10-12
35 (PC11-1)
36 (PC11-11)



N-Series

Log 12	1.079181246047624827722505692704101362736508627114 91294745072056255944553133251014201682285988 40
Ln 12	2.484906649788000310229709479838878840798490826543 259959976054352624281537157998398085342408806569 46387197299110709923720973904469744678093 5118
$\sqrt{12}$	3.464101615137754587054892683011744733885610507620 761256111613958903866033817600074162292373514497 1513512522828308134060599398901899979049 57623
$\sqrt[3]{12}$	2.289428485106663735616084423879354017831813841575 862144198104348131348598048428300875216322061834 09109741151880862991036403072245057723315 8752
$\sqrt[5]{12}$	1.643751829517225762308497936230979517383492589945 4752004110297675321076924920997929976201984189
$\sqrt[7]{12}$	1.426161635227378840484120685451442566729703987645 167174310576836788073683702545609558295152 3499
$\sqrt[10]{12}$	1.282088853986815440448530762915599482582088548947 0024248225064620936984144166070767860647648161
$\sqrt[100]{12}$	1.025160377800735874366696400689306837215424406711 7766369362591228717089052166108262799217452509
e^{12}	162754.791419003920808005204898486783170209284478 720770443556248138596770835543738729288241 909431684317816136420649516201423295328144
π^{12}	924269.181523374186222579170358475607172922268940 049306205759784137974139199236871159289927 187480032324169939326240538299381718549993
$\tan^{-1}$ 12	1.487655094906455389320653376988970144567453359059 533484252844829757910796806365896372280574782 0
12^{100}	828179745220145502584084235957368498016122811853 8 944354642018641032549193301212230377702832968580 1 9385573376

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